Домашно №3, Задача 1 по Алгебра 2 на Компютърни Науки

Камен Димитров Младенов, Група 6

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Решение

$$\frac{f(x)}{g(x)} = \frac{x^3 + x^2 - x + \bar{2}}{x^3 - \bar{2}x^2 - \bar{2}x + \bar{2}} \quad (x^3 + x^2 - x + \bar{2}) - (x^3 - \bar{2}x^2 - \bar{2}x + \bar{2}) = \bar{3}x^2 + x = r_1 \quad f(x) = g(x) + r_1$$

$$\frac{g(x)}{r_1} = \frac{x^3 - \bar{2}x^2 - \bar{2}x + \bar{2}}{\bar{3}x^2 + x} \quad (x^3 - \bar{2}x^2 - \bar{2}x + \bar{2}) - \bar{2}x(\bar{3}x^2 + x) = -\bar{4}x^2 - \bar{2}x + \bar{2} = x^2 + \bar{3}x + \bar{2}$$

$$(x^2 + \bar{3}x + \bar{2}) - \bar{2}(\bar{3}x^2 + x) = x + \bar{2} = r_2 \quad g(x) = (\bar{2}x + \bar{2})r_1 + r_2$$

$$\frac{r_1}{r_2} = \frac{\bar{3}x^2 + x}{x + \bar{2}} \quad (\bar{3}x^2 + x) - (\bar{3}x)(x + \bar{2}) = 0 \quad r_1 = (\bar{3}x)r_2$$

$$g(x) = \bar{2}(x + \bar{1})(\bar{3}x)r_2 + r_2 = (x^2 + x + \bar{1})r_2 \qquad f(x) = g(x) + (\bar{3}x)r_2 = (x^2 + \bar{4}x + \bar{1})r_2 \implies (f, g) = r_2$$

$$\begin{aligned}
&f(x) = g(x) + r_1 \iff r_1 = f(x) - g(x) \\
&\implies g(x) = \bar{2}(x + \bar{1})r_1 + r_2 = \bar{2}(x + \bar{1})(f(x) - g(x)) + r_2 \\
&\iff r_2 = \bar{2}(x + \bar{1})(g(x) - f(x)) + g(x) \\
&\iff r_2 = (\bar{2}x + \bar{2} + \bar{1})g(x) + (-\bar{2})(x + \bar{1})f(x) \\
&\iff r_2 = \bar{3}(x + \bar{1})f(x) + (\bar{2}x + \bar{3})g(x) \\
&\implies u(x) = \bar{3}x + \bar{3} \quad v(x) = \bar{2}x + \bar{3}
\end{aligned}$$